An Optimal Control Approach to Pilot/Vehicle Analysis and the Neal-Smith Criteria

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An optimal control pilot modeling technique is utilized in the conceptual framework of Neal and Smith who in 1970 presented a closed-loop frequency-domain technique for handling qualities analysis of highly augmented aircraft. By performing their analysis in the alternate manner demonstrated here, an attempt is made to bridge the gap between frequency-domain and time-domain approaches to pilot/vehicle analysis, to introduce what might be considered as a more general pilot modeling approach in this particular application, and thereby move towards extending this approach to even more complex vehicles and piloting tasks. Using the same quadratic cost weightings and set of observations for analyzing a variety of higher-order aircraft dynamics, the optimal control approach was successfully employed in the attempt to duplicate the Neal-Smith results. Specifically, it was found that the optimal control analysis resulted in pilot phase compensation for the configurations studied that were very comparable to those of Neal and Smith. In addition, the analysis here more closely reflected the actual inflight situation in that an actual tracking task was modeled. More importantly, the analysis demonstrated that low closed-loop bandwidth obtained from the analysis correlated well with bandwidth-related problems (i.e., large initial response delays) encountered in flight. This fact could be used as an additional criterion that must be met before considering the characteristics of resonance peak and pilot phase compensation.

Nomenclature

A, A_l, A_c, A_{veh} = plant matrices
b, b_l, b_{veh} = control vectors
Bw = bandwidth frequency
C_l, C_p = output matrices
d, d_c = disturbance vectors
$E\{\cdot\}$ = expectation operator
g = pilot weighting on input rate
$\bar{H}_p(s)$ = pilot transfer matrix
$H_{\theta_e}, H_{\dot{\theta}_e}, H_\theta, H_{\dot{\theta}}$ = dynamic elements of $\bar{H}_p(s)$
$H_a(s)$ = vehicle transfer function
J_p = pilot objective function
K_p = Neal-Smith pilot model gain
K_x = pilot control gain matrix
K_θ = vehicle (plant) gain
Q = pilot weighting matrix on outputs
r = pilot weighting on input
$S_{(\cdot)}(\omega)$ = spectral density of $(\cdot)$
t_{delay} = time delay
T_{p_1}, T_{p_2} = Neal-Smith pilot model lead and lag time constants
u_p = pilot's control input
v_u = pilot's neuromotor noise
$\bar{v}_y$ = pilot's observation noise vector
V_y = covariance of observation noise, $\bar{v}_y$
w = plant process driving noise
$\bar{x}, \bar{x}_c$ = state vectors
$\bar{y}$ = observation vector
$\zeta_{(\cdot)}$ = damping ratio of mode $(\cdot)$
θ = pitch attitude
θ_c = commanded pitch attitude
θ_e = pitch attitude tracking error $(\theta_c - \theta)$
$\sigma_{(\cdot)}$ = rms of $(\cdot)$
Σ_l = filter error covariance matrix (steady state)
$\tau_l, \tau_2, \tau_{\theta_2}$ = time constants of the flight control system and airframe
τ_N = pilot's neuromotor time constant
$\omega_{(\cdot)}$ = undamped natural frequency of mode $(\cdot)$
$\measuredangle(\cdot)$ = Bode phase angle of transfer funcjtion $(\cdot)$

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Introduction

WITH the advent of highly augmented aircraft, dynamic handling qualities specifications, based primarily on the conventional rigid-body modes, are no longer feasible. The augmentation's higher-order dynamics, for example, not only alter longitudinal short-period response, but also introduce additional modal dynamics in the vehicle response, seriously affecting pilot opinion.

To gain a better understanding of the effects of higher-order dynamics, as well as other aircraft characteristics, handling qualities researchers for several years have been engaged in closed-loop analyses of piloted vehicles. These analyses have relied heavily on analytical models of the pilot's loop closures, or pilot models. In general, pilot modeling has advanced under two categories: frequency domain or describing function models and time-domain or optimal control models (OCM). The former was extensively used by McRuer et al.[1] and others to mimic the characteristics of the human controller in a single-control-variable, single-display-variable, linear time-invariant system. This modeling approach has also been used (cf., Ref. 2) to investigate the correlation between pilot opinion and parameters of the classical pilot describing function in a single-axis tracking task. Later, McDonnell,[3] using a similar pilot model, showed that a large lead time constant in the pilot model is directly related to increased (degraded) pilot rating. He also demonstrated that a trend of increased (worsened) rating with increased closed-loop mean absolute error exists. Knowledge of these trends led Anderson[4] to create an analytical pilot rating prediction method for the two-axis hover task. In this approach, the parameters were chosen to minimize a pilot rating metric containing the two measures cited by McDonnell representing performance and workload, respectively. In each of these developments, the parameters of the classical describing function were used to explain pilot commentary.

Drawing upon these earlier findings, Neal and Smith[5] hypothesized that "pilot rating is a strong function of the pilot's compensation required to achieve good low-frequency performance and the pilot/vehicle oscillatory tendencies that resulted." Expressing good tracking performance in terms of closed-loop frequency response (Bode) characteristics, Neal and Smith devised a closed-loop analysis technique capable of exposing pilot problem areas in pitch attitude tracking. Furthermore, this technique was especially useful in evaluating high-order vehicle dynamic characteristics. The relative success of this method, still based on a single-loop, compensatory task description and a "controller" similar to the classical pilot describing function, raises the interesting question of whether these earlier results could be duplicated by using another pilot modeling approach that has the potential of extending the method, perhaps to more complex, multiloop tasks.

The optimal control approach to modeling the pilot loop closures, pioneered by Kleinman et al.[6] has also been successful in duplicating measured pilot describing functions and in matching mean square statistics. Furthermore, Hess,[7] followed by Schmidt,[8] have shown that with proper modeling the magnitude of the model's cost function correlates with subjective rating in a variety of tasks. Schmidt[9] has also used this correlation to propose a pilot-in-the-loop control augmentation synthesis technique.

What will be shown here is that such an optimal control modeling technique may be utilized in a Neal-Smith conceptual framework, and their analysis may thereby be performed in an alternate manner. By doing so, an attempt is made to bridge the gap between frequency-domain and time-domain approaches to pilot/vehicle analysis, to introduce what might be considered as a more general pilot modeling approach in this particular application, and thereby move towards extending this approach to even more complex vehicles and piloting tasks. To accomplish this requires a review of the Neal-Smith method with emphasis given to the implementation of the optimal control model (OCM) in the analysis. Next, the issue of modeling the tracking task used in the Neal-Smith flight experiments for use in the OCM will be addressed. Finally, the proposed OCM analysis and the results of re-evaluating some of the Neal-Smith aircraft configurations will be discussed.

Neal-Smith Methodology

Neal and Smith's investigation of the early 70's had a two-fold objective: to provide data on the effects of flight control system (FCS) dynamics and to develop a design criteria capable of pinpointing pilot problem areas encountered in performing a given task.

To meet the first objective a total of 51 basic FCS/short period configurations were flight tested. An overall pilot rating (Cooper-Harper)[10] representing a numerical summary of an aircraft's suitability to perform a given task was assigned to each configuration. Preliminary results concluded that the addition of FCS dynamics "can drastically alter the airplane's short period response."

Moreover, difficulties in using existing open-loop criteria to explain all the results of this experiment led to the development of an alternate approach; the "pilot-in-the-loop" analysis. Based on the assumption that the pilot's rating was primarily determined by how precisely the pilot could control pitch attitude, the analysis was conducted using the single-input, single-output compensatory tracking task modeled as in Fig. 1.

This diagram represents a compensatory tracking task where the pilot perceives only the difference between the aircraft's attitude and the command attitude. The pilot compensation is modeled to first order as a lead-lag with a time delay and gain. The pilot's time delay (0.3 s) includes the effects of perceptual delays and neuromuscular lags associated with the human operator.

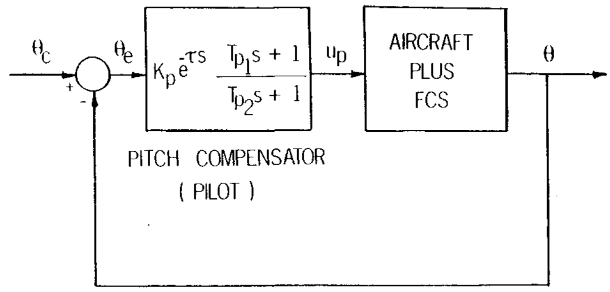

Fig. 1 Classical model structure.

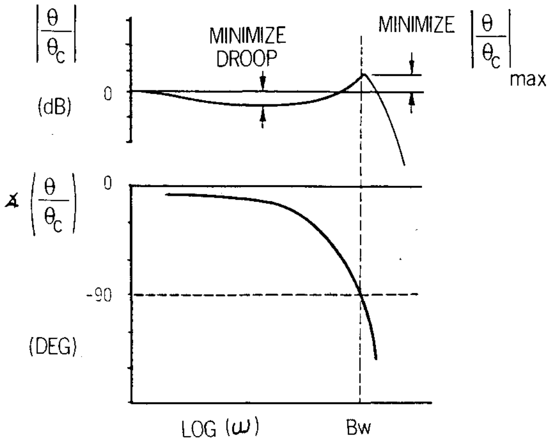

Fig. 2 Neal-Smith pilot strategy in tracking.

To aid in the coming discussion the following terminology should be noted:

θ/u_p = open-loop transfer function of the aircraft plus FCS

θ/θ_e = open-loop transfer function of the aircraft plus FCS plus pilot

θ/θ_c = closed-loop transfer function of the aircraft plus FCS plus pilot, which is related to θ/θ_e by Eq. (1)

$$\frac{\theta}{\theta_c} = \frac{(\theta/\theta_e)}{1 + (\theta/\theta_e)} \tag{1}$$

With the ultimate goal of evaluating the pilot and the aircraft engaged in a tracking task, Neal and Smith translated the pilot's tracking objective into closed-loop, frequency response specifications. (See Fig. 2.)

Clearly, the pilot wants to acquire the target quickly and predictably, with a minimum of overshoot and oscillation. The phrase "to acquire the target quickly and predictably" was interpreted to mean the pilot wants to attain a certain bandwidth, and below this frequency, keep the magnitude of (θ/θ_c) relatively close to 0 dB. Bandwidth (Bw) was defined as the frequency at which the closed-loop phase angle of (θ/θ_c) is -90 deg. Neal and Smith continued the interpretation of this phrase by correlating the desire "to minimize oscillation" with minimizing the closed-loop resonant peak $|\theta/\theta_c|_{max}$. They noted typically that pilot strategy was a tradeoff between striving for acceptable low-frequency performance and eliminating the accompanying oscillations.

The Neal-Smith investigation concluded that "pilot rating is a function of the compensation required to achieve good low-frequency performance and the oscillatory tendencies that result." They defined a measure of pilot compensation as the describing function's phase, exclusive of time delay effects, at the bandwidth frequency. Or,

$$\measuredangle_{pc} = \measuredangle \left(\frac{j\omega T_{p_1} + 1}{j\omega T_{p_2} + 1} \right)_{\omega = Bw} \tag{2}$$

This measure is frequently interpreted as being related to the pilot's physical and mental "workload."

Therefore, the analysis required the selection of the parameters (K_p, T_{p_1}, T_{p_2}) representing pilot compensation such that the following enumerated performance standards were met: 1) a bandwidth of 3.5 rad/s [The appropriate bandwidth selected for this task, and $\measuredangle(\theta/\theta_c) = -90$ deg defines the bandwidth frequency], 2) a maximum low-frequency droop of -3 dB (i.e., $|\theta/\theta_c| \geq -3$ dB for $\omega \leq Bw$); and the *type* of the compensation (or the ratio of T_{p_1}/T_{p_2}) led to a minimum value of resonant peak $|\theta/\theta_c|_{\max}$.

As shown in Fig. 3, Neal and Smith were able to correlate pilot rating with the resulting pilot compensation and magnitude of resonance peak. In the diagram, pilot ratings are divided into the three levels of handling qualities following the Cooper-Harper rating scale: level 1, 1.0-3.5—good; level 2, 3.5-6.5—fair; and level 3, 6.5-10.0—poor.

The following transfer function represents the vehicle dynamics of selected aircraft analyzed. (More were evaluated in Ref. 5.)

$$\frac{\theta(s)}{u_p(s)} = K_\theta (\tau_1 s + 1)(\tau_{\theta_2} s + 1) / \{ s(\tau_2 s + 1) [(s^2/\omega_3^2) + (2\zeta_3/\omega_3)s + 1] [(s^2/\omega_{sp}^2) + (2\zeta_{sp}/\omega_{sp})s + 1] \} \tag{3}$$

The corresponding parameters are listed in Table 1 according to the configuration number.

As an alternate technical approach, with the intent of extending the method to other tasks, as noted earlier, turn now to the incorporation of the OCM into the analysis.

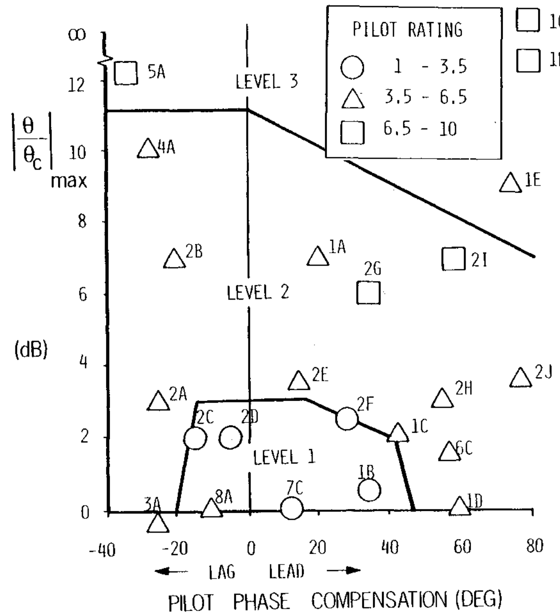

Fig. 3 Neal-Smith results.

Optimal Control Model (OCM)

The optimal control model (OCM)[6] of the pilot assumes that the well-trained, well-motivated human operator chooses his control input u_p subject to human limitations, such that an objective function of the following form is minimized

$$J_p = E\left\{ \lim_{T\to\infty} \frac{1}{T} \int_0^T (\bar{y}' Q \bar{y} + r u_p^2 + g \dot{u}_p^2)\, dt \right\} \tag{4}$$

Here, the weighting g is selected to obtain a chosen neuromuscular lag time constant τ_N, and the weightings Q and r are selected to reflect the objective in the task. The pilot's input is expressed as

$$\tau_N \dot{u}_p = -K_x \hat{x} - u_p + v_u \tag{5}$$

Table 1 Configuration summary

Configuration	$1/\tau_1$	$1/\tau_{\theta_2}$	$1/\tau_2$	ω_{sp}/ζ_{sp}	ω_3/ζ_3	Pilot rating
1A	0.5	1.25	2	2.2/0.69		4-6
1B	2.0	1.25	5.0	2.2/0.69		3.5
1C	2.0	1.25	5.0	2.2/0.69	16.0/0.75	3.5-5
1D	∞	1.25	½	2.2/0.69		3-5
1E	∞	1.25	5.0	2.2/0.69		6
1F	∞	1.25	2.0	2.2/0.69		8
1G	∞	1.25	0.5	2.2/0.69		8.5
2A	2.0	1.25	5.0	4.9/0.70		4.5
2B	2.0	1.25	5.0	4.9/0.70	16.0/0.75	4-6
2C	5.0	1.25	12.0	4.9/0.70		3
2D	∞	1.25	∞	4.9/0.70		2.5
2E	∞	1.25	12.0	4.9/0.70		4
2F	∞	1.25	5.0	4.9/0.70		3
2G	∞	1.25	5.0	4.9/0.70	16.0/0.75	7
2H	∞	1.25	2.0	4.9/0.70		5-6
2I	∞	1.25	2.0	4.9/0.70		8
2J	∞	1.25	0.5	4.9/0.70	16.0/0.75	6
3A	∞	1.25	∞	9.7/0.63		4-5
4A	∞	1.25	∞	5.0/0.28		5.5
5A	∞	1.25	½	5.1/0.18		5-7
6C	∞	2.4	∞	3.4/0.67		4
7C	∞	2.4	∞	7.3/0.73		1.5-4
8A	∞	2.4	∞	16.5/0.69		4-5

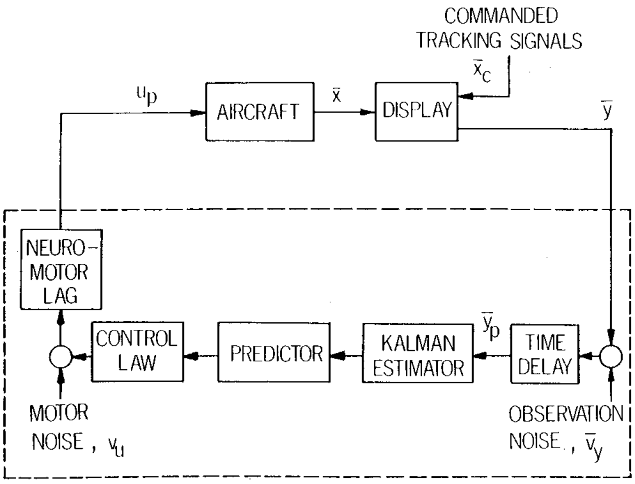

Fig. 4 Optimal control model.

Figure 4 gives a qualitative representation of the overall pilot model. A detailed discussion of the model can be found in Ref. 6.

To review briefly, the pilot model includes the pilot's observations $\bar{y}(t)$ modeled as a linear combination of delayed outputs corrupted by white measurement noise, or

$$\bar{y}(t) = C_p \bar{x}(t-\tau) + \bar{v}_y(t-\tau) \tag{6}$$

The state estimation is accomplished via a Kalman filter cascaded with a least-mean-square predictor. The optimal control gains are determined from the minimization of the cost function J_p, which represents the task description. As stated before, the first-order lag $(\tau_N s+1)^{-1}$ associated with the pilots neuromotor dynamics results naturally from including control rate $\dot{u}_p$ in the cost function J_p.

Consider now the consequences of minimizing tracking error $(\theta_c - \theta)$ by including it directly in the cost objective J_p. By neglecting (in this discussion temporarily) that portion of tracking error due to motor and observation noise (see Fig. 4), the mean squared value of error may be expressed as

$$\sigma^2_{\theta_e} = \frac{1}{\pi}\int_0^\infty \left| \frac{\theta_e}{\theta_c}(j\omega) \right|^2 S_{\theta_c}(\omega)\,\mathrm{d}\omega \tag{7}$$

where

$$\frac{\theta_e}{\theta_c} = 1 - \frac{\theta}{\theta_c}$$

and where S_{θ_c} is the spectral density of the commanded signal. S_{θ_c} defines the frequency content of θ_c and is related to the mean squared value of θ_c by

$$\sigma^2_{\theta_c} = \frac{1}{\pi}\int_0^\infty S_{\theta_c}(\omega)\,\mathrm{d}\omega \tag{8}$$

In considering Eq. (7), note that minimizing $\sigma^2_{\theta_e}$ forces the closed-loop system frequency response (θ/θ_c) to tend to unity over the frequency range of the command θ_c, with the lowest bound being attained by the ideal perfect tracker for which $|\theta/\theta_c| = 0$ dB *and* $\measuredangle\ (\theta/\theta_c) = 0$ deg across all (weighted) frequencies. The realization of the perfect tracker is unrealistic, however, since it implies an instantaneous transfer of energy between input and output. But, a tracker that *attempts* to be *ideal* across the weighted frequency band of the commanded signal is realizable.

Thus, the OCM by minimizing tracking error "automatically" minimizes the low-frequency droop and resonance peak of the closed-loop frequency response as in the Neal-Smith approach. Furthermore, as will be shown, the OCM will *automatically determine the closed-loop system bandwidth required* to best achieve the pilot's objective, given the limitations of the human and the characteristics of the subject vehicle. In other words, the pilot, as modeled by the OCM, is trying to achieve good low-frequency performance (a reasonable bandwidth with a minimum of low-frequency droop) plus good high-frequency stability ($|\theta/\theta_c|_{max}$ as small as possible).

In the next sections, the optimal control model will be incorporated into the pilot-in-the-loop analysis of Neal and Smith.

Synthesis of the Tracking Task via OCM

Three key elements concerning application of the pilot model must be established:

1) The pilot's observations and objective function to be minimized.

2) The system's representation in a tracking task.

3) A definition of the command signal to be tracked.

In this discussion, specific attention will be given to adapting the OCM to be consistent with the simplified compensatory tracking task model of Neal and Smith. But first, the appropriate objective function must be selected.

In the optimal control model's objective function

$$J_p = E\left\{ \lim_{T\to\infty} \frac{1}{T}\int_0^T (\bar{y}'Q\bar{y} + ru_p^2 + g\dot{u}_p^2)\,\mathrm{d}t \right\},$$

the weighting matrix, $Q = \mathrm{diag}[q_1, q_2, \ldots, q_m] \geq 0$, where m is the dimension of the observation vector $\bar{y}$; the weightings on control and control rate, (scalars in this analysis) $r \geq 0$ and $g > 0$; plus the elements of $\bar{y}$ must all be determined to quantify the task.

Obviously, the most critical parameter is the tracking error θ_e, the difference between the commanded attitude θ_c and the aircraft's attitude θ. Observation of attitude itself is also required if the task is one of pursuit rather than compensatory in nature. (A compensatory task is defined such that only error is observed.) In addition to θ_e and θ, studies[6] have shown that the human controller can also extract rate as well as position from a single display, thereby expanding $\bar{y}$ to

$$\bar{y}^T = (\theta_e, \dot{\theta}_e, \theta, \dot{\theta})$$

The next step in quantifying the pilot's control objective is selecting the cost function weightings. This is relatively simple in this case since the pilot is clearly performing attitude tracking. Previous investigations[11] have found that over a wide range of vehicle dynamics the following weights on θ_e, $\dot{\theta}_e$, and u_p

$$q_{\theta_e} = 16, \qquad q_{\dot{\theta}_e} = 1, \qquad r = 0$$

(with angles in degrees or radians if g is selected as discussed below) would accurately reflect the pilot's control objectives in the tracking task. To complete the definition of J_p the weighting on control rate must be set.

The weighting g on the control rate $\dot{u}_p$ is constrained by physiological limits. These limits are linked with the neuromuscular dynamics associated with the human controller modeled as a first-order lag. The associated lag time constant τ_N is expressed in the context of the pilot (model) control law,

$$\tau_N \dot{u}_p = -K_x \hat{\bar{x}} - u_p + v_u$$

For a given set of weights on $\bar{y}$ and u_p ($r = 0$ usually), adjusting g in the cost function determines τ_N. Finally, for aggressive control action, the lower limit on τ_N has been determined to be near 0.1 s,[1] based on experimental man-machine data.

The vehicle dynamics to be controlled must be represented by the linear time-invariant equations of motion:

$$\dot{\bar{x}}(t) = A\bar{x}(t) + bu_p(t) + dw(t) \tag{9}$$

To model the tracking task, the vehicle states must be augmented with the command signal states. The augmented system is structured as follows:

$$\begin{bmatrix} \dot{\bar{x}}_c \\ \dot{\bar{x}} \end{bmatrix} = \begin{bmatrix} A_c & 0 \\ 0 & A_{\text{veh}} \end{bmatrix} \begin{bmatrix} \bar{x}_c \\ \bar{x} \end{bmatrix} + \begin{bmatrix} 0 \\ b_{\text{veh}} \end{bmatrix} u_p + \begin{bmatrix} d_c \\ 0 \end{bmatrix} w$$

$$\bar{y} = \begin{bmatrix} I_2 & -I_2 & 0 \\ 0 & I_2 & \end{bmatrix} \begin{bmatrix} \bar{x}_c \\ \bar{x} \end{bmatrix} + \bar{v}_y \tag{10}$$

where the vehicle states are defined as $\bar{x}^T = [\theta, \dot{\theta}, u, \alpha, \text{etc.}]$, the command signal states $\bar{x}^T = [\theta_c, \dot{\theta}_c]$, and $I_{(\cdot)}$ indicates the identity matrix of appropriate dimension.

In this analysis the commanded attitude is generated by a second-order filter driven with white noise, or

$$\ddot{\theta}_c + 0.5\dot{\theta}_c + 0.25\theta_c = 0.25w(t) \tag{11}$$

where w is a zero mean, Gaussian white noise process. The statistics on θ_c and $\dot{\theta}_c$ [obtained with $\sigma_w^2 = 64\delta(t)$]

$$\sigma_{\theta_c} = 4 \text{ deg}, \qquad \sigma_{\dot{\theta}_c} = 2 \text{ deg/s}$$

indicate a reasonable, yet sufficiently challenging task to test pitch attitude tracking. Defining the commanded signal completes the objectives of this section. It should be emphasized that modeling the attitude tracking task as above approximates the *actual discrete instrument tracking task* used in the Neal-Smith flight experiments. Table 2 summarizes the resulting pilot model parameters as defined in Refs. 6 and 11.

The Methodology

The discussion turns now to the acquisition of those parameters required for the closed-loop analysis. Of course, the prerequisite for making any analysis comparable to that of Neal and Smith is the existence of a frequency-domain representation of both the controller and plant.

To obtain the closed-loop system frequency response, the coherent part of the pilot's control may be expanded in the frequency domain as

$$u_p(s) = H_{\theta_e}(s)\theta_e(s) + H_{\dot{\theta}_e}(s)\dot{\theta}_e(s) + H_\theta(s)\theta(s) + H_{\dot{\theta}}(s)\dot{\theta}(s)$$

Table 2 Baseline pilot model

Observation vector,	$\bar{y}^T = [\theta_e, \dot{\theta}_e, \theta, \dot{\theta}]$
Objective function weights,	$Q_{y_{ii}} = [16, 1, 0, 0]$, $r = 0$
Observation thresholds,	$T_{\theta_e} = T_\theta = 0.05$ deg
	$T_{\dot{\theta}_e} = T_{\dot{\theta}} = 0.18$ deg/s
Observation noise ratio,	$= -20$ dB
Fractional attention,	$f_i = 0.5$ all observed variables
Observation delay,	$t_{\text{delay}} = 0.2$ s
Neuromuscular lag,	$\tau_N = 0.1$ s
Motor noise variances,	-25 dB
Control input,	u_p (stick force)

where

$$\bar{H}_p(s) = [H_{\theta_e}(s), H_{\dot{\theta}_e}(s), H_\theta(s), H_{\dot{\theta}}(s)]$$

is obtained directly from the OCM[6] where

$$\bar{H}_p(s) = \frac{-\ell_e^*}{\tau_N s + 1}\left[(sI - \hat{A})\int_0^{t_{\text{delay}}} \exp(sI - A_1)\sigma d\sigma \times (sI - A_1 + b_1\ell_e^*) + sI - \hat{A} + b_1\ell_e^*\right]^{-1}\bar{\Sigma}_1 C_1^T V_y^{-1} \tag{12}$$

with

$\ell_e^* = [K_x \mid 0]$; pilot gains

$b_1 = \text{col}[0, \ldots 0, 1/\tau_N]$

$$A_1 = \begin{bmatrix} A & B \\ 0 & -\dfrac{1}{\tau_N} \end{bmatrix}; \text{ system matrices}$$

and

$C_1 = [C_p | 0]$ observation matrix

The steady-state estimation error covariance $\bar{\Sigma}_1$ is that for the Kalman filter, V_y the covariance associated with the pilot's observation noise, and $\hat{A}$ defined as

$$\hat{A} \doteq A_1 - \bar{\Sigma}_1 C_1' V_y^{-1} C_1$$

all defined in the OCM modeling approach. The previous expression for the pilot's control input may then be rewritten as

$$u_p(s) = [H_{\theta_e}(s) + sH_{\dot{\theta}_e}(s)]\theta_e(s) + [H_\theta(s) + sH_{\dot{\theta}}(s)]\theta(s) \tag{13}$$

where the bracketed terms are the equivalent transfer functions relating tracking error and attitude angle to pilot input, respectively. Given the aircraft attitude transfer function as

$$\theta(s) = H_a(s)u_p(s) \tag{14}$$

and

$$\theta_e(s) \doteq \theta_c(s) - \theta(s)$$

the desired closed-loop transfer function is simply

$$\frac{\theta(s)}{\theta_c(s)} = \frac{H_a(s)[H_{\theta_e} + sH_{\dot{\theta}_e}]}{1 + H_a(s)[(H_{\theta_e} + sH_{\dot{\theta}_e}) - (H_\theta + sH_{\dot{\theta}})]} \tag{15}$$

A block diagram of this θ/θ_c transfer function is presented in Fig. 5.

In the compensatory tracking model used by Neal and Smith, however, the inner loop $(H_\theta + sH_{\dot{\theta}})$ was not included. Its contribution to the pilot's compensation thereby was assumed small. (Since the actual task was one of pursuit, this was clearly an approximation.) If the pilot transfer function here is taken simply as $H_p(s) = H_{\theta_e} + sH_{\dot{\theta}_e}$, the approximate closed-loop transfer function is, of course

$$\frac{\theta(s)}{\theta_c(s)} = \frac{H_a(s)H_p(s)}{1 + H_a(s)H_p(s)} \tag{16}$$

identical to the Neal-Smith compensatory tracking model. Finally, note that Neal and Smith did not directly include noises, or remnant, in their analysis. But the transfer func-

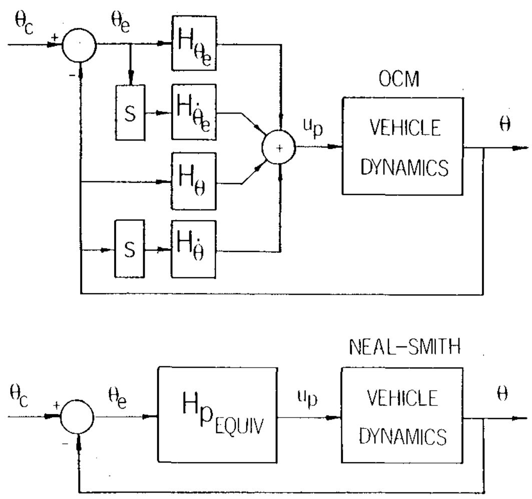

Fig. 5 Model schematic comparison.

tions obtained here reflect the presence of motor and observation noise in the system (as in Fig. 4), but of course do not *directly* include noise in them.

Now the closed-loop frequency response ($s=j\omega$) is reduced to algebraically manipulating the elements of $\bar{H}_p(j\omega)$, along with the vehicle's $H_a(j\omega)$.

To estimate the effect of ignoring the inner loop in Fig. 5, construct the spectral densities of the system's variables θ_c, θ_e, u_p, and θ using the simpler model structure. The spectral density is related to the respective mean squared value by the relation

$$\sigma_x^2 = \frac{1}{\pi}\int_0^\infty S_x(\omega)\,\mathrm{d}\omega \tag{17}$$

if x is zero mean process. Since θ_c, θ_e, u_p, and θ are zero mean processes, the above integration will yield the system variances $\sigma_{\theta_c}^2$, $\sigma_{\theta_e}^2$, $\sigma_{u_p}^2$, and σ_θ^2. These variances then may be compared to their counterparts obtained from the complete OCM's state covariance matrix. A comparison of the two results should indicate the agreement between the complete model and this simplified model.

The method for calculating the needed spectral densities, assuming θ_c as the only source (i.e., ignoring remnant), proceeds as follows:

$$S_{\theta_c}(\omega) = |H_I(j\omega)|^2 S_w(\omega)$$

$$S_{\theta_e}(\omega) = \left|\frac{\theta_e}{\theta_c}(j\omega)\right|^2 S_{\theta_c}(\omega)$$

$$S_{u_p}(\omega) = \left|\frac{u_p}{\theta_e}(j\omega)\right|^2 S_{\theta_e}(\omega)$$

$$S_\theta(\omega) = \left|\frac{\theta}{u_p}(j\omega)\right|^2 S_{u_p}(\omega) \tag{18}$$

where S_w is the process noise intensity with $H_I(s)$ the commanded signal shaping filter's transfer function. Once the spectral densities are evaluated over a sufficient band (ω), the integration can be performed numerically to obtain the desired variances. A sample of these variances and their complete OCM-derived counterparts are presented in Table 3.

Surprisingly, the effect of neglecting the inner attitude loop, and neglecting the control and measurement noises made no substantial impact on the system's variances here. Thus, the simpler closed-loop structure is a close approximation to the OCM in this case. On the other hand, it is evident that the OCM is an appropriate model for the Neal-

Table 3 Variance comparison for configuration 2A

	Reduced system	Actual, OCM
$\sigma_{\theta_c}^2$, deg^2	15.55	16.00
$\sigma_{\theta_e}^2$, deg^2	0.68	0.65
$\sigma_{u_p}^2$, lb^2	5.24	6.23
σ_θ^2, deg^2	13.47	14.75

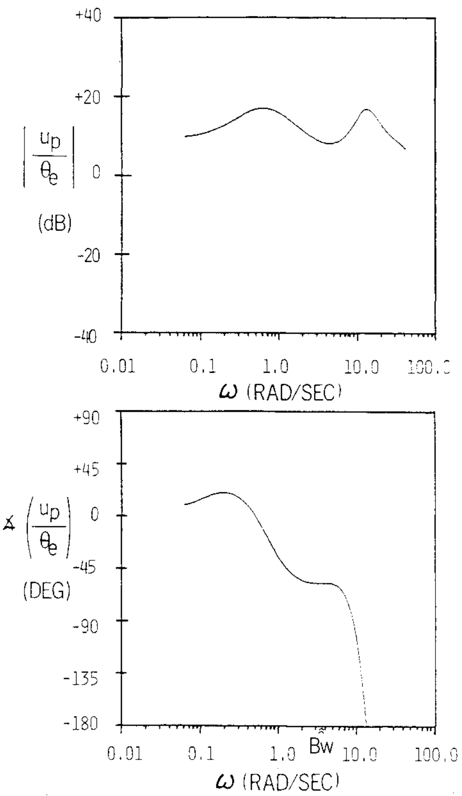

Fig. 6 Configuration 2D pilot frequency response.

Smith analysis. It remains to identify those required parameters deemed critical to their pilot/vehicle analysis.

Since the frequency responses (Bode plots) of both the pilot control model (OCM) and the closed-loop pilot/vehicle system are now available, the measures of bandwidth, pilot phase compensation, and resonance peak are obtained directly from these frequency responses. These measures have the same definitions used by Neal and Smith; however, they are no longer obtained by specifying bandwidth and "droop" a priori. As an example, consider Figs. 6 and 7, resulting from the analysis of Neal and Smith's configuration 2D. The controller (pilot) in configuration 2D is trying to minimize the objective function, subject to the incorporated *human* limitations (i.e., $t_{\text{delay}}=0.2$ s, $\tau_N=0.1$ s). Hence the resulting combination of bandwidth, droop, and phase compensation is a product of modeling configuration 2D directly in the given tracking task.

To reiterate, first consider the bandwidth. The definition remains unchanged; bandwidth is the frequency at which the closed loop $\measuredangle(\theta/\theta_c)$ equals -90 deg. Unlike Neal and Smith, bandwidth is now the outcome of the analysis directly, dependent on task, vehicle, and human factors. In the OCM, the neuromuscular lag τ_N replaces bandwidth as the fundamental specified quantity, and is chosen to reflect aggressive tracking behavior. From Eq. (12) τ_N's effect on the pilot's characteristics is self-evident, but it is not-so-self-evident effect on closed-loop bandwidth (speed of response) that is of interest. As τ_N is increased [by selecting a larger

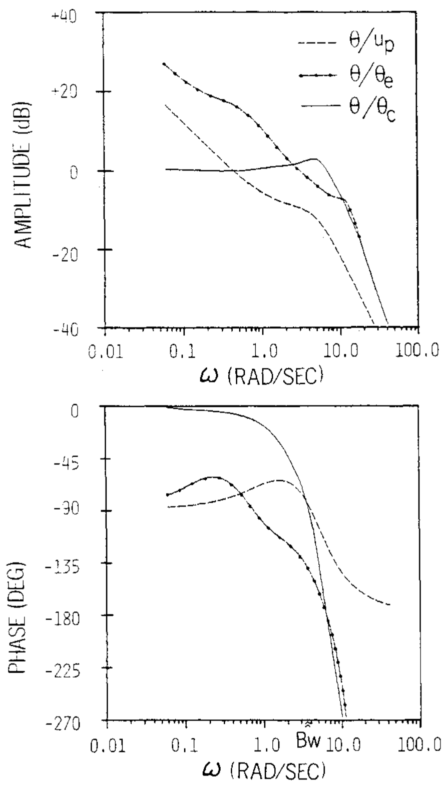

Fig. 7 Configuration 2D system frequency response.

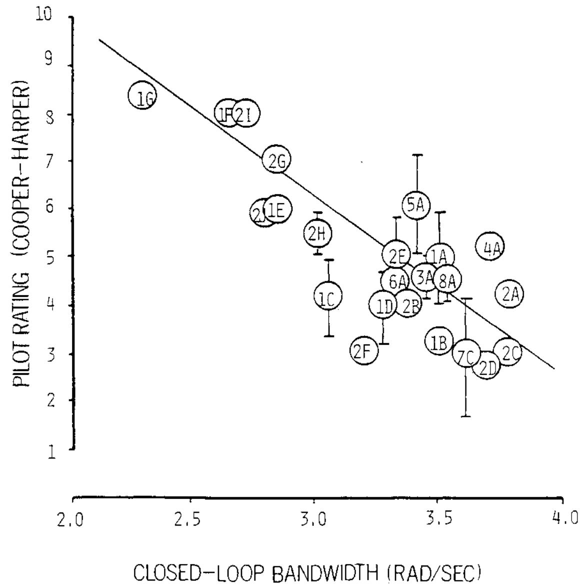

Fig. 8 Rating/bandwidth correlation.

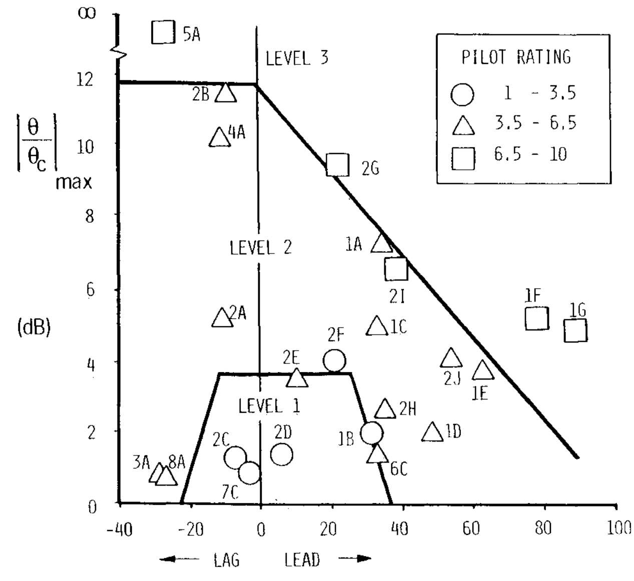

Fig. 9 Results of optimal control analysis.

value for g in Eq. (4), for example], the *closed-loop* bandwidth decreases. Therefore, "relaxed" pilot behavior, or large τ_N, is associated with a closed-loop system exhibiting slow response characteristics. Conversely, aggressive pilot behavior, or a low τ_N, produces a higher bandwidth, producing a more responsive and more aggressive pilot/aircraft combination. Since the minimum τ_N is usually accepted to be 0.1 s for aggressive tracking, setting this value in the OCM determines the *maximum achievable bandwidth* for the system. It is interesting to note that the resulting bandwidth for configuration 2D above was 3.7 rad/s.

The second measure is the pilot phase compensation. The total pilot phase compensation from the OCM is the phase angle of the pilot's frequency response evaluated at the system bandwidth frequency ($\omega = Bw$) This compensation, however, includes the effects of neuromuscular lag τ_N and the perceptual time delay t_{delay}. To be compatible with Neal-Smith, these effects may be adjusted via the following expression

$$\measuredangle_{pc} = \measuredangle(\text{OCM})_{\omega = Bw} + 57.3\, t_{\text{delay}} Bw + \tan^{-1}(\tau_N Bw) \tag{19}$$

where $\measuredangle_{pc}$ corresponds to Neal and Smith's interpretation of pilot phase compensation. For the purpose of correlation, Neal and Smith could have included their (constant) effective time delay as part of the pilot phase compensation. And since their bandwidth was fixed, this would simply slide the phase compensation scale (see Fig. 3) lower by a fixed angle for all aircraft configurations.

The last measure, magnitude of resonance peak, $|\theta/\theta_c|$ is available from the closed-loop system's Bode plot, and subsequently will be discussed further. Hence all the quantities are obtainable through the OCM modeling process presented in this section.

Results

Application of the method to several Neal-Smith aircraft configurations, chosen on the basis of their levels of handling qualities, resulted in the pilot rating/bandwidth correlations of Fig. 8. These results reveal a trend of degraded pilot opinion with decreased closed-loop bandwidth, particularly where $Bw \leq 3$ rad/s in this task. This result is in complete agreement with Neal and Smith who found that the required bandwidth in this task was approximately 3.5 rad/s. Pilot comments mention an inability to track without several pilot-induced oscillation (PIO) problems in configurations 1G, 1F, 2I, and 2G, in support of the above results. Pilots also complained that these aircraft were "real sleepers," i.e., they had large initial response delays, another indication of insufficient bandwidth.

The question of how to expose poor handling when a sufficient bandwidth is present is related to the results of Fig. 9, in which pilot rating is shown to depend on closed-loop resonance peak and pilot's phase compensation. But how to obtain these results requires further discussion. With sufficient closed-loop bandwidth, large time delays of the initial responses are no longer a problem. The problem, which incidentally is evident in the rms tracking errors, is rooted in the tradeoff between errors due to low-frequency vs high-frequency performance. Generally, when faced with an aircraft exhibiting some PIO tendencies, the pilot will "back off" and sacrifice low-frequency performance in order to minimize rms error due to any excessive resonance in the system. This statement was justified by comparison between the maximum low-frequency "droop" obtained in this analysis and the pilot's comments on the subject aircraft response. Upon comparing cases with similar bandwidth frequencies, the ones found to exhibit a lower droop automatically had higher analytically obtained rms track errors, along with associated pilot comments indicating some overshoot and PIO problems. This "delicate" strategy of avoiding lightly damped oscillations (or PIO's) may be interpreted in terms of the OCM as a desire for a stable optimal solution, or the OCM will always result in the "best" achievable piloting strategy. Alternatively, one could argue that PIO's are the results of a suboptimal piloting strategy, and in particular, the pilot's "gain" is too high (above the "optimal" predicted by the OCM).

Now based on this hypothesis, suppose the gain is increased above that of the OCM in the "forward path," e.g., to raise the closed-loop droop to try to achieve a higher level of tracking performance. Typically, the droop of the configurations analyzed ranged from -0.3 to -1.0 dB. Adjusting the additional forward path gain to achieve

$$|\theta/\theta_c| > -0.6 \text{ dB for } \omega \leq Bw \tag{20}$$

produced resonant peaks comparable to those of Neal and Smith and therefore exposed those PIO prone configurations with this parameter. This procedure may sound reminiscent of the original Neal-Smith approach, but only one degree of freedom exists—that of "DC" or constant gain adjustment. This gain adjustment was accomplished as an integral part of the comptuer-based analysis in the following way.

1) Scan the magnitude of the closed-loop droop for minimum $|\theta/\theta_c|_{\omega=\omega_{\min}}$; $0<\omega_{\min}<Bw$ for $\omega_{\min}$ (frequency at the droop).

2) At $\omega_{\min}$ record the open-loop $(\theta/\theta_e)(s)$ pilot frequency response as the complex number $\alpha+i\beta$.

3) Find additional pilot gain K to produce -0.6 dB droop at $\omega_{\min}$ for the closed-loop frequency response.

4) Add K to the open-loop frequency response $(\theta/\theta_e)(s)$ and re-evaluate the closed-loop frequency response

$$\frac{\theta}{\theta_c}(s) = \frac{(\theta/\theta_e)(s)}{1+(\theta/\theta_e)(s)}\bigg|_{s=j\omega}$$

5) Check droop and scan for the corrected $|\theta/\theta_c|_{\max}$.

As an example, consider Figs. 10 and 11 illustrating configuration 2G before and after the adjustment of the feedforward gain. In spite of 2G's poor rating of 7 and pilot commentary indicating PIO problems, the "optimal" OCM solution, represented in Fig. 10, yielded a relatively low resonance peak of 4.38 dB. The solution does show an unacceptable bandwidth of 2.85 rad/s and a low droop of -0.74 dB. Using the method outlined above raises the droop to -0.060 dB (and the bandwidth to 3.30 rad/s). More importantly, the magnitude of resonance peak in Fig. 11 (9.25 dB) is significantly increased due to the increase in feedforward gain (of 1.73 dB). The total pilot phase compensation (-25.67 deg) at the original bandwith frequency is obtained from the pilot's open-loop frequency response (Fig. 12), and is corrected via Eq. (19) to yield the lead of 22.97 deg depicted in Fig. 9.

Some interesting results from using the OCM approach presented here are evident in the results of Fig. 9. Similar to the analysis of Neal and Smith, the grid of Fig. 9 is separated into levels 1, 2, and 3, signifying good, fair, and poor handling qualities, respectively. Of particular interest here are those vehicles the Neal-Smith approach failed to place in the correct "areas." In one instance, configuration 8A appeared in the level 1 area, although it received a level 2 rating. The OCM not only identified it properly as level 2, but placed the

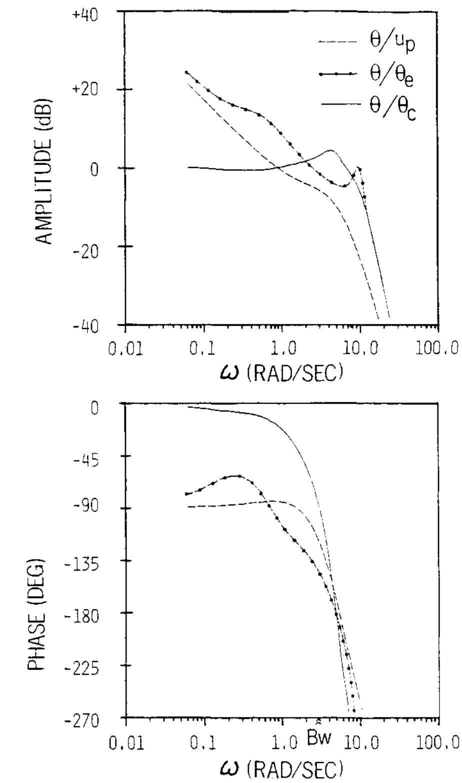

Fig. 10 Configuration 2G system frequency response.

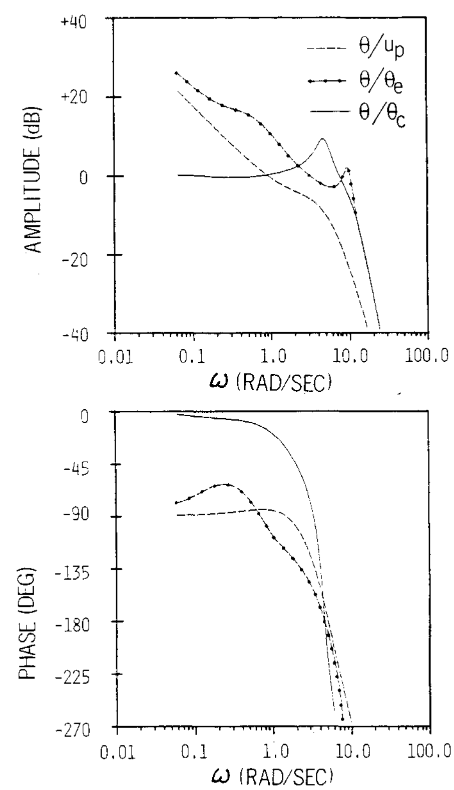

Fig. 11 Configuration 2G adjusted system frequency response.

aircraft next to another configuration (3A) with different short period characteristics, but sharing the same pilot comments and pilot ratings (4-5). Other examples, such as level 3 configuration 2G, were incorrectly placed in the level 2 area by Neal-Smith. Once again, the OCM approach predicted a PIO problem serious enough to warrant a level 3 rating. In cases correctly rated by Neal and Smith, agreement

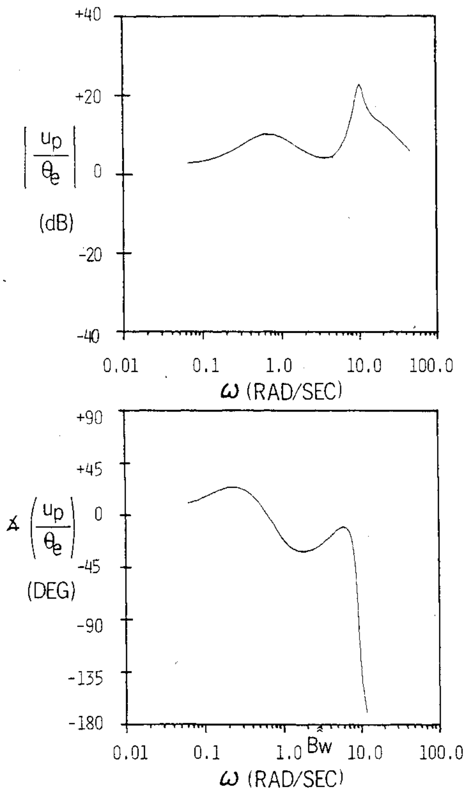

Fig. 12 Configuration 2G pilot frequency response.

was almost always attained by the OCM approach. Configurations 2C, 7C, and 2D were all given high marks in acquiring the target, which exemplifies the level 1 rating predicted by both methods. In only two cases evaluated (configurations 2E and 2F) did the OCM method yield marginal results. These configurations are on the level 1/level 2 boundary, and only one rating data point was obtained for each configuration.

Finally, it should be noted that for virtually all of the pilot/vehicle combinations addressed, the slope of the open-loop (pilot plus aircraft plus FCS) magnitude was -20 dB/decade at the crossover frequency (frequency at which the magnitude is 0 dB). This can be observed, for example, in Figs. 7 and 10, corresponding to configurations 2D and 2G evaluated here. Therefore, this analysis technique is also consistent with this important characteristic that forms the basis of McRuer's crossover modeling approach.[1]

Conclusion

Of the original 51 configurations evaluated by Neal and Smith, 23 were chosen for re-examination on the basis of spanning the three levels of handling qualities in the earlier effort. Using the same weightings and set of observations for analyzing this variety of aircraft for the given task, the optimal control model was employed in the attempt to duplicate the Neal-Smith results. This, in fact, was accomplished. Specifically, it was found that the optimal control model analysis resulted in pilot phase compensation for the configurations studied that were very comparable to those of Neal and Smith. In addition, the analysis here more closely reflected the actual inflight situation, in that an actual inflight tracking task was modeled. More importantly, the analysis demonstrated that low closed-loop bandwidth obtained from the analysis correlated well with bandwidth-related problems (i.e., large initial response delays) encountered in flight. This, in fact, could be used as an additional criterion that must be met before considering the other characteristics. Consequently, the initial findings cited were encouraging; however, the fact that the optimal control model would not produce the similar closed-loop resonance peaks, indicative of pilot-induced oscillation tendencies, required further analysis.

To expose the potential for the oscillatory tendencies experienced in flight by the pilot, a "DC" gain increase was added to the optimal control model results, and resonance peak-to-gain sensitivity was examined. This characteristic might be considered as a substitute for the resonance peak itself as a key fundamental parameter. Just how valid this step is relies on its underlying assumption that a tradeoff exists between low-frequency performance and minimizing oscillatory tendencies, and that the pilot-induced oscillation tendencies might be minimized by optimal pilot equalization. This assumption can only be scrutinized with more piloted simulations over a wide range of FCS/aircraft dynamics.

Acknowledgments

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